البحث السادس

SMU
SOUTHERN METHODIST UNIVERRSITY

Report project for:

QUEUING THEORY

EMIS 7370: Probability and Statistics for Scientists and Engineers

Fall 2014

Professor Dr.

Jerrell Stracener

Done by:

Wafaa Alshehri

41128923

Table of contents

Abstract

The queuing theory deals with the most undesirable lines on our life Undesirable. We experience the queues in several parts on our life. For instance, we get into line at bank, supermarkets, salons, petrol stations, and public departments, and waste many minuets or can be hours by only waiting to be served. Moreover, there are several invisible queues that we see them only from the software and hardware system such as request by user into web service get into the queue until finish. The goals of this paper to know how implementing of the queuing theory in multiple places could increase the efficiency of the work system that can effect positively the profits by decreasing the waiting time in the queues.

Introduction

Historically, the queuing theory was raised by Agenr Krarup Erlang when he treated congestion problems in the beginning of 20th century. His work was an inspiration to many engineers to implement the queuing theory to deal with the queuing problems .After that results of the queuing theory have been used in many sides such as operation research, traffic engineering, and computer science. The organization that care of business profits and clients can be provided as queuing theory. The queuing theory is the mathematical study of reducing waiting lines. We can predict the time and the length of queues by using the queuing theory. Hence, using the queuing theory has become fundamental part of many business sectors. Considerable body of entrepreneurs or the responsible of the work system have shown that queuing theory can be useful in real business world by setting the balance between the capacity problems against the productivity and services. Moreover, it manages losing sales, customers, and money due to the waiting time.

Reducing the waiting time, and providing appropriate service to clients in short period are important elements that should be considered for any service provider. In many stores and public department, management has tried to minimize the frustration of customers by increasing the speed of the checkout and cashier lines. How to implement the queuing theory? There are two elements to deal with. They are service time and arrivals rate. Each of those elements must be collected and applied to the theory. The theory aims to generate a simple model that helps maximize service time and increase customer satisfaction. This paper will apply the queuing theory to development the system work, the capacity of production, and increasing the profits as an attempt to make it better and better. Thus, we need model to analyze such situations.

Objective

The main goal of this project is to understand the queuing theory, and how we can apply it in business sector. In addition, we will provide an example of the bank to help them to know the number of employees that they should have in order to be competitive while retaining a low-cost work structure. It will also calculate the expected waiting times of the customers, providing a base model that could be extrapolated into other similar situations.

Background

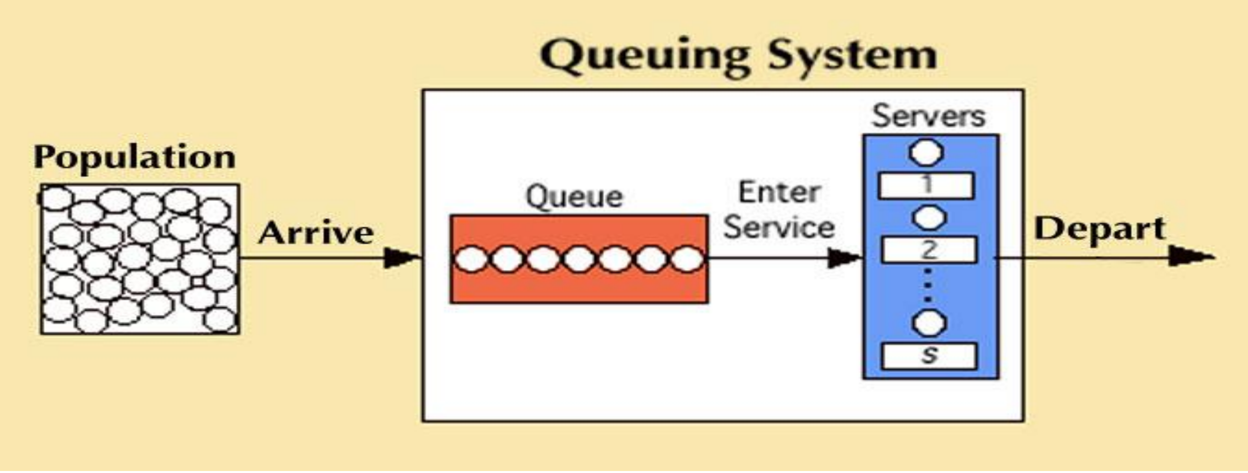
The present report serves as an example of the real-life application of Queuing Theory. Artificial data resembling the waiting times at the service halls in a bank is used for the purposes of the analysis. Servicing customers properly is the main target for any institution, especially for the financial institutions. They have entire queue management systems which are normally based on the Queuing Theory. As dealing with the formation of the queues, the aim of the models based on the Queuing theory is predicting the queue lengths and waiting times based on some historical data to derive the needed estimations from. The purpose of this example is to develop a model which can be used as a tool for finding the optimal number of servers given the input parameters characterizing the customers' inflow.

Queuing Theory

It is mathematical study for waiting line (queue). This theory tends to reducing time and line. It's common on hospitals, bank, office, and shops.

Queue and System:

Figure 1- queue and system



Queuing Examples: "General"

Queuing system	Customers	servers
Bank	Bank customers	Computers , tellers
airport	airplanes	Gates , runways
Grocery store	Grocery customers	Clerk with registers

Table-1 *Queuing examples*

Multi-server configurations:

Single Queue

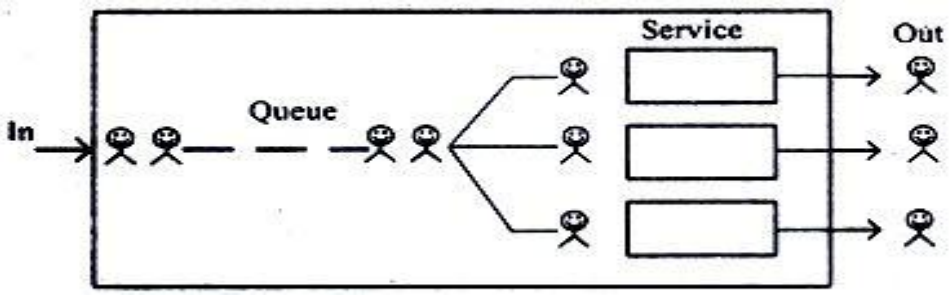


Figure 2- single queue

Multi Queue:

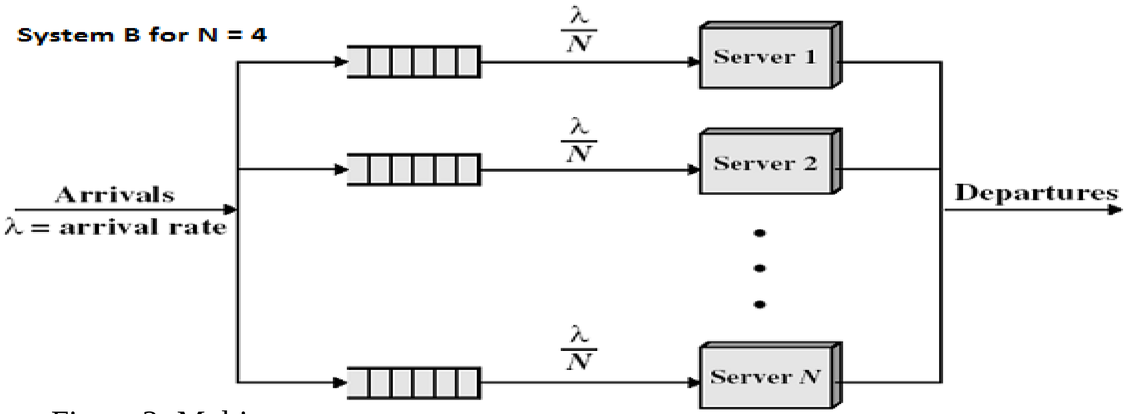


Figure 3- Multi queue

Characters of Queuing Theory:

1-Arriving Populations

Population (sizes) :

1*Finite in number, Example, machines needing repair

2*infinite, Example, hospital patients.

2- Arrival rate: λ

*constant: often generated by a machine.

*variable, random:

Follow some probability dist. (Poisson and exponential distribution)

λ : Average no. arrivals per unit time T

Arrivals per time: Poisson distribution.

Time between arrivals: negative exponential distribution.

3- Service rate: μ

μ= average no. arrivals served per time T

***Kendal-Lee notation:** a/b/s

a: arrival process: M= Poisson

B: service process: M= exponential, $A(t) = 1 - e^{-\lambda t}$

D=deterministic. "The value is constant from distribution"

S, number of parallel servers.

Service facility structure:

* single-channel, 1-phase

* single-channel, multi-phase

* Multi-channel, single-phase

* multi-channel, multi-phase

Queue discipline:

* Firs-come, First-served (FCFS)

* Last-come, first-served

* Priorities (importance rankings)

* Earliest due date

Example Data

The artificial data representing a queue of customers in the banking service halls. The number of customers arriving was generated via algorithm in Excel as a Poisson random number generated series with a parameter sλ= 10 clients/hour.

Descriptive statistics

Mean	10
Standard Error	0.20

Median	9
Mode	7
Standard Deviation	3.16
Sample Variance	10.01
Kurtosis	0.36
Skewness	0.50
Range	19
Minimum	2
Maximum	21
Sum	2295
Count	240

Table 2- *Descriptive statistics*

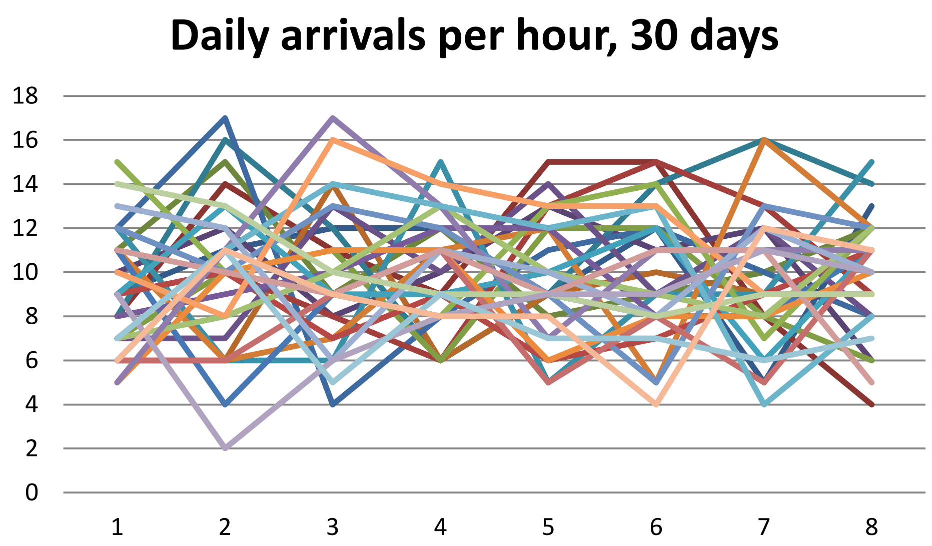


Figure 4- daily arrivals per hours

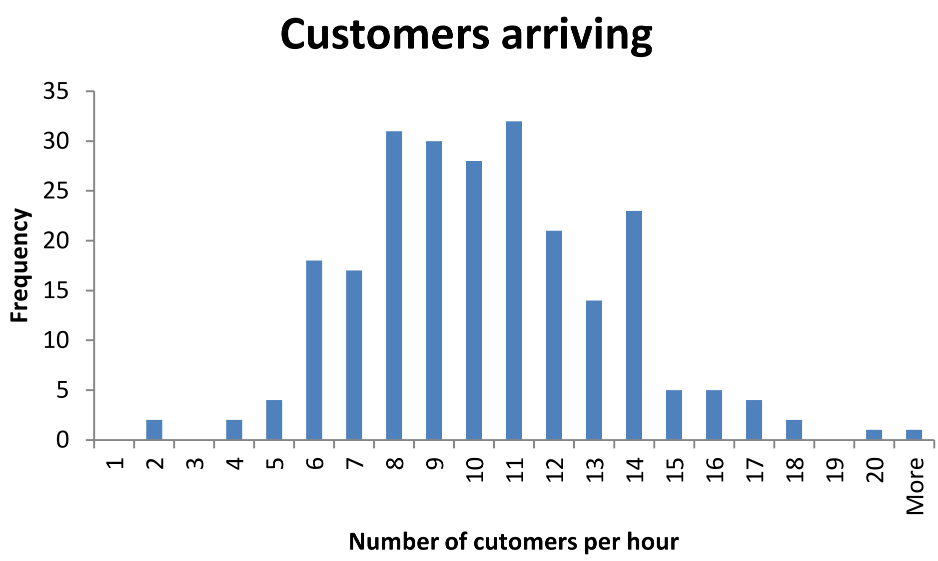


Figure 5- customers arriving

Methodology

The Queuing Theory with a multi-server configuration will be used. The following set-up is valid:

- Infinite size of the arriving population: customers of the bank with the corresponding arrival rate λ which will further be estimated based on the available data. The random variable λ is assumed to follow a Poisson distribution and therefore its characteristics will be applied in the parameter estimation.

- The service facility structure is single-channel with 1 phase.

- The service rate, µ is the average number of arrivals who are serviced per hour. Rather than being estimated via the Exponential distribution for example, its value is deterministic,µ**= 5 clients/hour**

- The number of servers, s will enter the calculations as changing vales from 1 to 7 in order to find the optimal number of servers given the characteristics of the queue.

Therefore the initial inputs in the system are the following:

Parameter	Value
Λ	10
M	5
S	1 to 7

The above input values serve for deriving the estimates of:

- the expected fraction of time the individual servers are busy, known as the utilization factor (ρ)
- the probabilities that there are 0 customers at any given time (P(0))
- the expected number of customers in queue at any given time (L_q)
- the expected number of customers in system at any given time (Ls)
- the expected waiting time in queue (W_q)
- the expected waiting time in the system (W_s)

The formulas needed to derive the above estimates are provided in the table below. After that a table with the obtained results for each of the 1 to 7 servers cases.

Formulation:

Variable	Description	Formula
ρ	Utilization Factor	$\rho = \frac{\lambda}{s\mu}$
P_0	Probability of 0 customers at any given time	$P_0 = \frac{1}{\sum_{n=0}^{s-1}\left(\frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}\right) + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!\,(1-\rho)}}$
L_q	Expected number of customers in queue at any given time	$L_q = \frac{\rho\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\,(1-\rho)^2}$
L_s	Expected number of customers in system at any given time	$L_s = L_q + \frac{\lambda}{\mu}$
W_q	Expected waiting time in queue	$W_q = \frac{L_q}{\lambda}$
W_s	Expected waiting time in system	$W_s = \frac{L_s}{\lambda}$

Table 3 - *Formulas for Queuing Theory parameters*

Following the formulas in the above table and considering the input values of the average number of customers per hour and the service rate a set of calculated parameters was obtained for 7 possible situations: each with number of available servers from 1 to 7.

Rationale

The relationship between the expected number of customers in queue and in the system and the expected waiting time in the queue and in the system is supported by Little's formula. It states that in the steady-state queuing process the expected number of customers waiting equals the expected waiting time multiplied by the lambda parameter of the Poisson distribution which is taken as the mean values of the number of arriving customers.

The waiting time in the system is naturally given by the waiting time in queue + the average servicing time.

While the number of customers arriving is given by the Poisson distribution with a location parameter λ, $\frac{1}{\lambda}$ is the expected inter-arrival time.

Calculation:

Parameter	Number of servers						
	1	2	3	4	5	6	7
λ	200%	100%	66.67%	50%	40%	33.33%	28.57%
P(0)	0	0.5	0.125	0.15	0.155	0.156	0.1564
L_q	∞	0	1	0.1666	0.1377	0.0104	0.0022
L_s	∞	2	3	2.1666	2.1377	2.0104	2.0022
W_q	∞	0	0.1	0.0166	0.0137	0.0010	0.0002
W_s	∞	0.2	0.3	0.2166	0.2137	0.201	0.2002

Table4-

EstimatedParameters

Results

According to the results if there is only one server, the system will not be able to exist because the number of customers arriving: 10 per hour will be higher than the server can be handle: 5 per hour. That is as well presented by the parameter utilization factor, which takes value of 200% in that case which means the fraction of time the individual server is busy will be 200% which is outside anyone's limit. After exploring the above table with results of the analysis it is evident that the minimum number of servers that are needed to serve the clients is three. If there are only two servers in the system they will be busy 100% of the time, e.g. the system will be performing at full capacity. However, when the system has three servers, it will be occupied at 66.67% of the time. However, it would still be expected that there will be around 1 client per hour in the queue and 3 clients waiting in the system. For four servers the expected number of clients on the queue will be 1/6 or 1 client for 6 hours while it would be expected that 2.166 customers will be waiting in the system or 13 clients in 6 hours.

The other important result is the waiting time. For a 3-serversystem the waiting time in queue will be 0.1 hour, which is 6 minutes and the waiting time in the system is 0.3 hours which is 18 minutes. The two types of waiting times aredecreasing the more servers are introduced into the system to nearly no waiting time in queue in case there are 7 servers (0.0002 hours which is approximately 0.72 seconds).

Although there is not great difference between the utilization factor for 3-server and 4-server system: 66.67% against 50% there is big difference in the waiting times in queue between the two set-ups. For a 4-server system set up it is 0.0166 hours ~1 minute and for a 3-server system set up it is 0.1 hour ~ 6 minutes. That is a decrease of 5 minutes waiting time for a “loss” of 16.67% (the difference between 66.67% and 50%) utilization. However, that as well means that the 4-server system can manage to service better any additional customer that arrives due to the 50% free resources. The difference between a 4-server and a 5-server setup with regard to waiting time is very small and is not worth adding one additional employee.

The probability of no customers at any given time in any case where the servers are more than 2 is increasing the number of servers increases. That is an expected effect since the more servers there are, the more availability to service the coming customers there is and respectively the number of waiting customers decreases. The expected number of waiting customers, both in queue and in the system in general is as well directly related to the probability of exact number of customers.

The probabilities for more than one client for each of the 7 settings of the system are derived following that formula:

$$P_n = \frac{(\frac{\lambda}{\mu})^n}{n!} * P_0$$

The probability of exactly 0 clients are obtained in Table 4 above and will be used for deriving the probabilities for more than 0 clients.

	Number of servers						
	1	2	3	4	5	6	7
P(0)	0	0.5	0.125	0.15	0.155	0.156	0.1564
P(1)	0	1	0.25	0.3	0.31	0.312	0.3128
P(n), n≥2	0	$\frac{2^n * 0.5}{n!}$	$\frac{2^n * 0.125}{n!}$	$\frac{2^n * 0.15}{n!}$	$\frac{2^n * 0.155}{n!}$	$\frac{2^n * 0.156}{n!}$	$\frac{2^n * 0.1564}{n!}$

Table 5- *Probabilities for number of customer*

Conclusion

The results show that the optimal system set-up is with 4 servers. Adding more than that number of employees is not conventional with regard to the very small added value for the waiting time. However, if cost analysis is as well included the optimal solution of that example might change. However, the purpose of that example were limited to deriving the components of a queuing system: waiting times, expected number of customers and service times using the Queuing Theory.

References

1. "Queueing Theory." Wikipedia. Wikimedia Foundation, 23 June 2014.
2. "Little's Law as Viewed on Its 50th Anniversary", Little, J. D. C.(2011), Operations Research
3. "Little’s Law: The Science Behind Proper Staffing", Harris, Mark (February 22, 2010).
4. 2011 International Conference On Management And Artificial Intelligence.Case Study for Restaurant Queuing Model
5. "Queue." Answers. Answers Corporation, n.d. Web. 04 July 2014.
6. "What People Hate Most About Waiting in Line." Slate Magazine, 2014.
7. "Queueing: Theory and Practice." Nature 179.4557 (1957): 459-60.

/https://isindexing.com/isi

Show
entries
Search:

ournals	Impact Factor	ISSN	Publisher Name
ITERNATIONAL JOURNAL OF DUCATIONAL AND SYCHOLOGICAL SCIENCES	2018 **0.915** (2018)	2536-9261	Arab Organization for Scientific Research and Human Development
ournals	**Impact Factor**	**ISSN**	**Publisher Name**

Showing 1 to 1 of 1 entries
Previous1Next

International Journal of Educational and Psychological Sciences

E-ISSN	2536-9261
P-ISSN	
Publisher Name	Arab Organization for Scientific Research and Human Development
Email	communication555@yahoo.com
Language	Arabic - English
Starting Year	2016
Discipline	The type of font in the text of the Arabic research using the traditional Arabic font size (16), the headings PT Bold Heading (14) Bold, the sub-headings (14) Bold, with margins of each size (3.5 cm left and right) 3.5) cm up and down the page). Lea
Frequency	Monthly
Website	http://www.ijeps.com
Country	Afghanistan
Accessing Method	Egypt Governorate of ELSahrqia Province of Abuhamad
Articles Format	PDF
License Type	Priority
Impact Factor	***0.915 (2018)***

Description

International Journal , Arab, National , Specialized , Scientific and Periodical : Arab International Specialized Scientific Magazine, Scientific Court, issued by the Arab Organization for Scientific Research and Human Development No. 3406 for the year 2016 in the Arab Republic of Egypthttp://derp.sti.sci.eg/details.aspx?Id=3490, the International Journal of Specialized Qualitative Research, accredited by the Supreme Council of Egyptian Universities(180-2018) . The research is judged by some professors of the Supreme Council of Egyptian Universities, and some professors of Arab and international universities such as Ohio University in the United States of America for foreign research, and supervises the study of the Arab influence coefficient. Issued by an international editorial board and a panel Specialized education, and taking into account the criteria of the promotion committees in the dissemination of modern research and in cooperation with senior professors of education and specializes in the research of the faculties of "Specific Education - Kindergarten - Early Childhood - Physical Education - Art Education - Music Education - Home Economics - Fine Arts, Technology and Development - Agricultural Economics The magazine is published quarterly (January, April, July and October) every year, and distributed by the Arab House for Training and Publishing. The magazine is concerned with the publication of everything new and modern in the field of studies and research. Arabic In addition, the magazine publishes research in Arabic and foreign languages such as English, French, German and Italian. The studies and research published in the magazine are subject to scientific arbitration. Flour with specialized professors.